Cosmic string in the BTZ Black Hole background with time-dependant tension

J. Sadeghi ^{a*}, H. M.Farahani^{a} †and S. M. Noorbakhsh^{a‡}

^{a}*Sciences Faculty, Department of Physics, Mazandaran University,*

P .O .Box 47415-416, Babolsar, Iran

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Abstract

In this paper we study the circular loops equation with time-dependant tension in the BTZ black hole background. We obtain various cases that loops of cosmic string finally collapse to form black holes. Also we study the effect of the BTZ black hole mass and angular momentum on the evolution of cosmic string loops. We find the critical values of initial radii as a limit for the cosmic string loops collapsing to form black holes.

1 Introduction

As we know the interaction between two fields plays an important role in cosmology and particle physics [1]. The connecting link was provided by Kirzhnits [2] who suggested that spontaneously broken symmetries can be restored at sufficiently high temperatures. According to modern ideas, the elementary particle interactions are described by a grand unified theory (GUT) with a simple gauge group G which is a valid symmetry at highest energies. As the energy is lowered, the model undergoes a series of spontaneous symmetry breaking [1]. On the other hand in hot big bang cosmology this spontaneous symmetry breaking implies a sequence of phase transitions [3]. The phase transitions in the early universe can give rise to topological stable defects like strings. A discussion of their evolution were given by Kibble [4]. Cosmic strings are one of the topological spacetime defects and are hypothesized to form when the field undergoes a phase change in different regions of spacetime. This is resulting in condensations of energy density at the boundaries between regions.These cosmic strings can lead to very interesting cosmological consequences such as explaining galaxy formation

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*Email: pouriya@ipm.ir

†Email: h.farahani@umz.ac.ir

‡Email: s.m.noorbakhsh@umz.ac.ir

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[5-7]. Because of their spacetime metric with a deficit angle they can produce a number of distinctive and unique observational effects. Also Cosmic strings can be produced at the end of inflation or at the end of brain inflation [8,9] providing us with a potential window on M theory [10-12]. However, the CMB rules out cosmic strings as seeds of large scale structure formation in the universe because of limits on their tension ($G\mu < 10^{-6}$) [13-15]. Furthermore cosmic strings still have strong influence on astrophysics [16-18] such as gravitational lensing effects [19, 20], gravitational wave background [21, 22] and early reionization [23, 24]. They would have immense density and also would represent significant gravitational sources. A cosmic string 1.6 kilometers in length would exert more gravity than the Earth. Cosmic strings would form a network of loops in the early universe and their gravity could have been responsible for the original clumping of matter into galactic superclusters. After the formation of cosmic string , they are not static and would continuously evolve under the force of their tension. They can collide and intersect to undergo reconnections also strings stretch under the influence of the Hubble expansion or the environment. In that case the strings lose energy to gravitational radiation when they oscillate. The reconnections of long strings and large loops will produce small loops copiously [25]. The observational results to support the existence of cosmic string loops in our Universe are in [20,26]. A cosmic string's vibration, which would oscillate near the speed of light, can cause parts of the string to pinch off into an isolated loop. These loops have a finite lifetime due to decay via gravitational radiation. Gravitational lensing of a galaxy by a straight section of a cosmic string would produce two identical undistorted images of the galaxy. A gravitational lens called CSL-1 which invokes two images with comparable magnitudes of the same giant elliptical galaxy was discovered. It is interesting that many similar objects were found in Ref.s [19, 27]. In 2003, a group led by Mikhail Sazhin reported the accidental discovery of two seemingly identical galaxies very close together in the sky, leading to the speculation that a cosmic string had been found [28]. However, observations by the Hubble Space Telescope in January 2005 showed them to be a pair of similar galaxies, not two images of the same galaxy [29,30]. A cosmic string would produce a similar duplicate image of fluctuations in the cosmic microwave background, which might be detectable by the upcoming Planck Surveyor mission. As a complicated time dependent gravitational source, the cosmic string loops oscillate with time rather randomly. Schild et al observed the brightness fluctuation in the multiple-image lens system Q0957+561A, B [20,26]. They think that system consists of two quasar images separated by approximately 6 degrees. The phenomena are images of the same quasar because of the same spectroscopic , fluctuating in brightness and time delay between fluctuation sit which is due to lensing that they supply this synchronous variations in two images. The cosmic string loops can also generate more gravitational waves and distinct signatures [21, 22, 31], nearly all loops will become black holes except in special cases. In Minkowski spacetime and Robertson-Walker universe, the loops will collapse to form black holes under their own tension certainly instead of remaining oscillating loops after the loops formation [32, 33]. In de Sitter backgrounds, only loops with large initial radii can avoid becoming black holes [32, 34, 35]. However a large loop will evolve to be a lot of smaller loops, loops can not live in a de Sitter spacetime unless they are very large. Clearly in the environment with a positive cosmological constant few cosmic string loops can survive. The black holes

are final results for loops of cosmic string if the tension of cosmic string is constant [36]. So far in nearly all researches considered the tensions of cosmic strings to be constant which is just an assumption but M. Yamaguchi put forward the important issue that the tensions of cosmic strings can depend on the cosmic time [37,38]. As we know in context of cosmological phase transitions when the field theory under symmetry group G as $U(1)$ is broken the cosmic strings are formed. So, we consider a Higgs field ϕ with following self -interaction potential,

$$V(\phi) = \frac{1}{2}\lambda(\phi^{\dagger}\phi - \eta^2), \tag{1}$$

where ϕ is a complex field and $\lambda > 0$. When symmetry is broken the vacuum expectation value of ϕ is $<\phi>= \eta$ and the tension of cosmic strings μ is given by $\mu \simeq \eta^2$ (μ is tension of cosmic string). In the high temperature limit we have an additional potential,

$$V_T(\phi) = AT^2(\phi^{\dagger}\phi + V(\phi)). \tag{2}$$

As we see the effective mass of the field $\phi \propto \eta^2$ thus η depends on the temperature T and hence on the cosmic time η is $\eta = \eta(t)$ [1]. This implies that the tension μ also depends on the cosmic time $\mu(t)$[37]. Since η is associated with scale factor $a(t)$ then the tension can be denoted as $\mu \propto a(t)^{-3}$ that $a(t)$ is the scale factor. As we know $a(t) \propto t^{\frac{2}{3}}$ in a matter dominated universe and also $a(t) \propto t^{\frac{1}{2}}$ in a radiation dominated universe. So the tension become proportional to $t^{\frac{2}{3}}$ and $t^{\frac{1}{2}}$ respectively. We have a general case of power -law expansion :$a(t) \propto t^{\alpha}$ [1]. The results show that time varying tensions of cosmic strings can cause matter power spectra and CMB completely different from those induced by constant tensions [37,38].

In order to explain cosmic string dynamic we need to find the Numbu- Gouta action.In that case we choose ξ^0, ξ^1 parameters as time like and space like coordinates. If we choose the gauge condition on string world-sheet coordinates we may us the conformal gauge where $\dot{x}^2 + x'^2 = 1$, so we will obtain wave equations as $\ddot{x}^{\mu} + x''^{\mu} = 0$ or $\ddot{x}^{\mu} - x''^{\mu} = 0$ [39,40]. In this case the motion of the string must be periodic in time with the period , $T = \frac{M}{2\mu}$ where M is the mass of the loop [41]. In this paper we consider the wave equation as $\ddot{x}^{\mu} - x''^{\mu} = 0$. All above information give us motivation to consider circular loop with time dependent tension.and study the equation of cosmic string loops in Banados , Teitelboim and Zanelli (BTZ) black hole background. Also we investigate several cases of the BTZ black hole with circular loop information.

2   The BTZ black hole metric

As we know the black hole solution of BTZ in (2+1) space time are derived by the following three dimensional theory of gravity,

$$S = \int d^3x \sqrt{-g}(^{(3)}R + \Lambda), \tag{3}$$

The BTZ black hole is obtained as the quotient space $SL(2,R) < (\rho_l, \rho_R) >$, where $< (\rho_l, \rho_R) >$ denote the subgroup of $SL(2,R) \times SL(2,R)$ generated by (ρ_l, ρ_R). In the coordinate chosen here the metric reads

$$dS^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)dt^2 + \frac{dr^2}{(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2})} + r^2(d\phi - \frac{J}{2r^2}dt)^2, \qquad (4)$$

where M is Arnowwitt-Deser-Misner (ADM) mass , and J is angular momentum (spin) of the BTZ black hole where $-\infty < t < \infty$,$0 \leq R < \infty$ and $0 < \theta < 2\pi$ with the cosmological constant $\Lambda = \frac{1}{l^2} > 0$

The metric (2) is singular when $r = r_\pm$

$$r_\pm^2 = \frac{Ml^2}{2}(1 \pm [1 - (\frac{J}{Ml})^2]^{\frac{1}{2}}), \qquad (5)$$

and

$$M = \frac{r_+^2 + r_-^2}{l^2}, J = \frac{2r_+r_-}{l}. \qquad (6)$$

Here we emphasis that if $|J| > Ml$,$r\pm$ become complex and the horizon will disappear. In case of $M = 1$, $J = 0$ the metric may be recognized as the ordinary AdS space.

3   The circular loop equation

Now we study behavior of a circular cosmic string in the BTZ black hole background (4). A free string propagating in a spacetime sweeps out a world sheet which is a two-dimensional surface. The Nambu-Goto action for a cosmic string with time-dependent tension is given by

$$S = -\int d^2\sigma\mu(\tau) \left[(\frac{\partial x}{\partial\sigma^0}\frac{\partial x}{\partial\sigma^1})^2 - (\frac{\partial x}{\partial\sigma^0})^2(\frac{\partial x}{\partial\sigma^1})^2\right]^{\frac{1}{2}}, \qquad (7)$$

where $\mu(t)$ is the string tension which is function of cosmic time. Here $\sigma^a = (t, \phi)(a = 0, 1)$ are timelike and spacelike string coordinates respectively. $x^\mu(t, \phi)(\mu, \nu = 0, 1, 2, 3)$are the coordinates of the string world sheet in the spacetime. For simplicity and without generality we assume that the string lies in the hypersurface $\theta = \frac{\pi}{2}$ then the spacetime coordinates of the world sheet parametrized by $\sigma^0 = t$,$\sigma^1 = \phi$ can be selected as $x = (t, r(t, \phi), \frac{\pi}{2}, \phi)$. In the case of planar circular loops, we have $r = r(t)$. According to the metric (4) and the d spacetime coordinates mentioned above, the Nambu-Goto action with an additional factor for the time-dependent tension belonging to a cosmic string denoted as

$$S = -\int dtd\phi\mu(t)r(h - \frac{\dot{r}^2}{h})^{\frac{1}{2}}, \qquad (8)$$

where $h = (-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2})$. Let $l = 1$, then we introduce the following equation for loops which have time-varying tension as $\mu(t) = \mu_0 t^q$.

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$$h^2\ddot{r}r + \frac{q}{t}r\dot{r}(h^2 - \dot{r}^2) - \dot{r}^2(h^2 + 3r^2h) + h^4 + r^2h^3 - \frac{J^2}{4r^2}(h^3 - 3\dot{r}^2) = 0, \qquad (9)$$

where μ_0 is a constant. Recently people have solved the equation of circular loop cosmic string for the Minkowski spacetime, Robertson-Walker universe, de-Sitter spacetime and Keer-de Sitter spacetime [17, 33, 34, 36, 42, 43]. They have found some constraints that a loop of cosmic string should not collapse to form of black hole for the case of constant and time-dependent tension. And time-dependent tension changing as the power q like $\mu(t) = \mu_0 t^q$ in different spacetimes. For example Vilenkin in Ref.[17] shown that in case of Minkowski and FRW spacetime with tension cosmic string constant all closed loops finally will be collapse to form of black hole. Then Cheng and Liu [2] studied equation of circular loops of cosmic string with time-dependent tension in the Minkowski and FRW spacetime and shown that there must exist a critical value $q_f = -0.131$. In the Minkowski spacetime when $q < q_f$ all cosmic string loops will expand to evolve or contrarily will collapse to form black holes when $q > q_f$. And for FRW background metric shown that there also exist a special value for every era, $q_r = -0.078$ for radiation-dominated era and $q_m = -0.062$ for matter-dominated era. In each era when $q < q_r$ or $q < q_m$ all cosmic string loops will expand to evolve with any values of $r(t_0) = r_0$ the initial.

So Larsen [34] solved equation of loops in de-Sitter spacetime with constant tension, it was shown that loops of cosmic string will keep on expanding in de Sitter spacetimes if their radii satisfies the condition like $r(t_0) > 0.707L$. After that Cheng and Liu [36] solved equation of loops in de-sitter spacetime with time-dependent tension. They have shown that the results same as Larsen [34] when the initial radius is $r(t_0) > 0.707L$. But in the case of $r(t_0) < 0.707L$ there must exist a critical value denoted as α, when $q < \alpha$, the cosmic string loops will enlarge to evolve or contrarily will collapse to form black holes when $q > \alpha$.

Now we are going to solve equation (9) numerically for the case of $q = 0$ and $q \neq 0$. In that case we account M and J as variables, we find constraint for initial radii of cosmic string loops collapsing to form black holes or vice versa will expand to evolve. First we suppose that tension as constant $q = 0$, $J = 0$ and $M = 1$, we get the same result as de-Sitter spacetime solved by Larsen[34]. In that case for collapsing to form black holes we have $r(t_0) < .7070L$. And in case of $J \neq 0$ and $M = 1$ the results are different and they are classified in the following tables (1).

J	$Critical\ r(t_0)$
-3	$r(t_0) > 4.80L$
-2	$r(t_0) > 3.740L$
-1	$r(t_0) > .707L$
$-2/3$	$.357L < r(t_0) < .707L$
-1/2	$.26L < r(t_0) < .707L$
0	$r(t_0) < .707L$
1/2	$.26L < r(t_0) < .707L$
2/3	$.357L < r(t_0) < .707L$
1	$r(t_0) > .707L$
2	$r(t_0) > 3.740L$
3	$r(t_0) > 4.80L$

Table 1: $M = 1, q = 0$ and different J.

In table (1) we present the some critical initial radii of the loops which lead to form of black holes. It is completely obvious in the case of $|J| \geq 1$, it should be $r(t_0) > \alpha$ that α is the critical value of radii, we see that by increasing positive and negative $|J|$ the critical initial radii become larger and we see same symmetry in values of J and radii.And if $|J| < 1$ not only satisfy to the above information but also has an extra constraint for initial radii as $\alpha < r(t_0) < .707L$ ($r(t_0)$ is the initial radii). We see in table(1),results for $|J| > ML$ and $|J| < ML$ are different. $|J| > ML$,($M = 1$) implies that how much the angular momentum of the black hole increases, only cosmic loops of larger radii can form black holes and all the cosmic loops having smaller radii than critical initial radii will survive. Since $0.707L$ is a very big amount and to the size of the universe then most of the loops will keep on expanding and the black hole formation possibility will decline.We see in figure(1), $|J| > 1$ the critical value of initial radii is larger than L which means that $r(t_0)$ is larger than the size of the universe so their existence in such condition may be impossible.
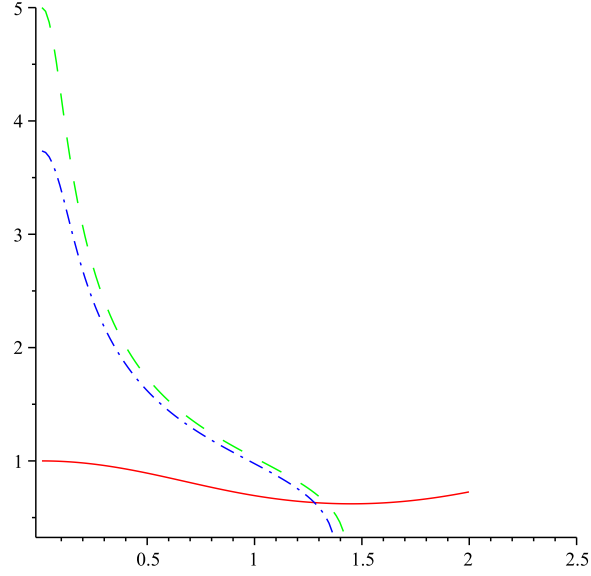
Figure 1: Graph for the $r(t)$ radii of circular loops in the BTZ black hole with $M = 1, J = 2$. The solid, dash-doted and dashed curves with initial $r(t_0) = 1.00$,3.740,5.00 respectively under $\dot{r} = 0$ and $(q = 0)$.

But we see in figure (2), $|J| < Ml$,$(M = 1)$ the result is completely different. Here loops of larger than $0.707L$ will remain in our universe and smaller ones will collapse to form black holes then there will be more possibility of forming black holes.
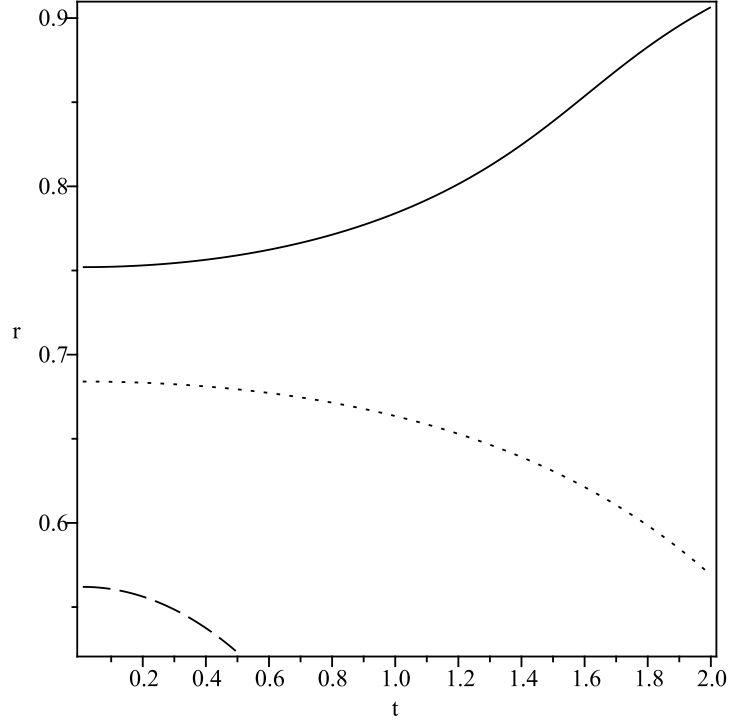
Figure 2: Graph for the r(t)radii of circular loops in the BTZ black hole with $M = 1, J = .5$.
The solid ,dash and doted curves with initial radii
$r(t_0) = .752, .562, .684$ respectively under $\dot{r} = 0$ and(q=0).

For other values of mass as a constant and different J results are the same and the only difference is in their critical initial radii.

Now we take J as a constant and M is different. so we find some constraint on the initial radii which give us the formation of black holes.

M	$Critical\ r(t_0)$
$For M < 0$	Any initial radii
M=0	Any initial radii
1	$r(t_0) > .707L$
2	$r(t_0) > .369L$
3	$r(t_0) > .294L$
10	$r(t_0) > .160L$

Table (2). Results For $J = 1, q = 0$,when M is different.

As explained in the table(2), when the mass of the black hole is $M \leq 0$ there is no constraint on the initial radii of the cosmic string loops for collapsing to form black holes.

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In that case there is no chance for loops to survive in the space time and variation of J has no affect to avoid the collapsing of the loops become black hole. When the mass of the BTZ black hole is $M > 0$ then the loops for collapsing should satisfy constraint as $r(t_0) > \alpha$ and the values of α decrease with respect to increase of the BTZ mass. We find that by increasing the mass of BTZ black hole the less loops will stay because the initial radii is small, so there are more loops satisfying this condition. Here, we must point out for the spinless and $M = 1$ BTZ black holes which we have to the de-Sitter spacetime which solved by [34]. In the special case ,he has shown for initial radii $r(t_0) < 0.7070L$ the loops will collapse to form black hole.But we showed that the BTZ black hole with angular momentum as $J \geq 1$,the result is clearly contrast whit his result. It means that the loops with condition like $r(t_0) > 0.7070L$ will collapse to form black hole, and also we note that smaller loops avoid to become black hole.The observational results show that there could exist a lot of cosmic string loops in our universe [26, 44] .The final results of the cosmic string loops in the BTZ background in case of $J > 1$ lead us to have lots of loops.
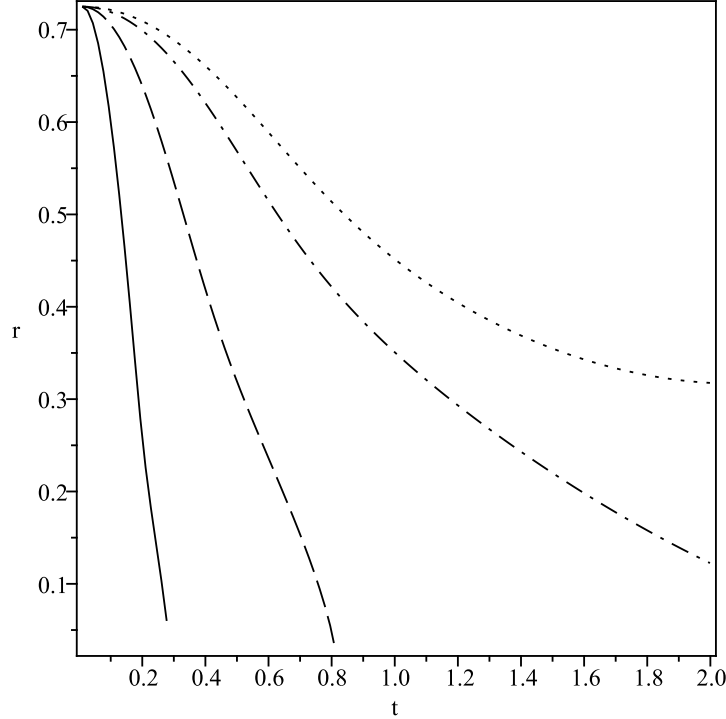

Figure 3: Graph for the $r(t)$ radii of circular loops in the BTZ black hole with $J = 1$. The solid ,dash and doted curves with $M = -3.8, -0.85, 0.002, .23$ respectively and initial value $r(t_0) = 0.725L$ and $\dot{r} = 0$ and q=0

Fig.(3) shows that if the magnitude of the BTZ black hole mass decreases, then the rate of collapsing loops become faster.

Next, we shall discuss about the cosmic strings with time-varying tension. For the loops of cosmic string as the power of time$\mu = \mu_0 t^q$ we solve equation (7) numerically.As above mentioned the values of large J can not case collapsing loops to black hole,Now we consider $J = 1$ and $M = 1$ and $\mu = \mu_0 t^q$, and we find that for all values of $q \geq 0$ our results are same

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as $q = 0$. It means that the loops satisfying $r(t_0) > .707L$ finally collapse to black holes,and the value of $q > 0$ can only has influence on the time of collapsing see fig(4).
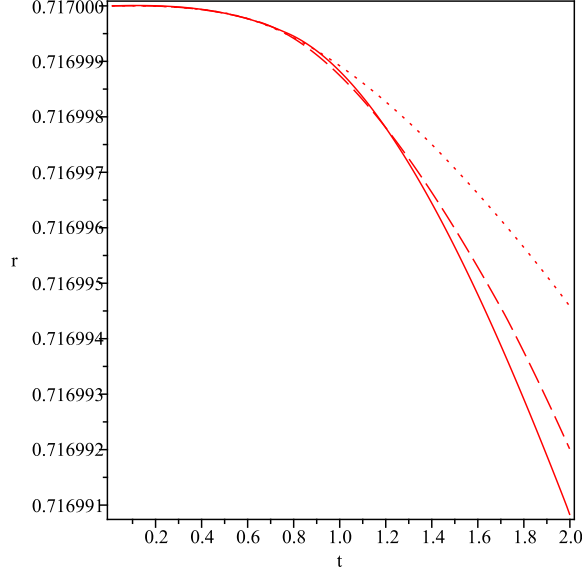

Figure 4: The solid, dash and doted curves of r(t) with $q = 1.5, 2.3, 4.5$ respectively. $M = 1$, $J = 1$. initial value $r(t_0) = 0.717L$ and $\dot{r} = 0$

Fig.(4) show that by increasing the values of q the loops take longer time to become black holes.

But for $q < 0$, when $r(t_0) > .707L$, we can find two limits for q as, $q < -4.78$ and $-.65 < q < 0$ where loops avoid forming black holes, that is clearly shown in figure(5).
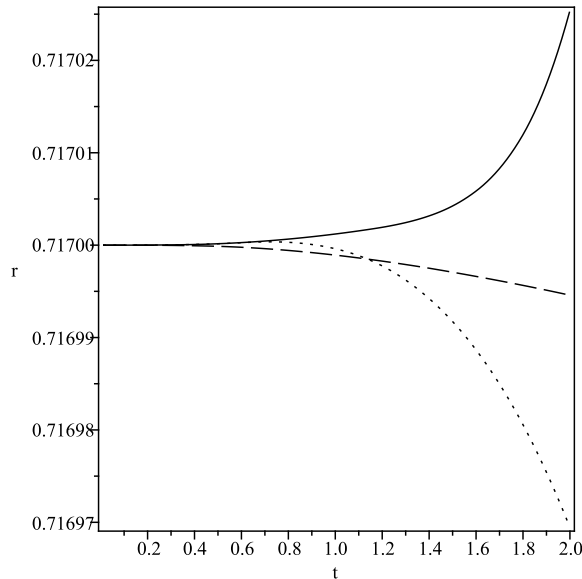

Figure 5: The solid, dash and doted curves of r(t)as functions of cosmic time with $q = -7.9, 4.5, -2.3$ respectively. in case of $M = 1$, $J = 1$ and initial value $r(t_0) = 0.717L$ and $\dot{r} = 0$

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4 Conclusion

In this paper we solved equation of a circular loop cosmic string in the BTZ background, that evolving in the hypersurface with $\theta = \frac{\pi}{2}$. By solving equation (7) numerically ,we try to obtain various cases that loops of cosmic string finally change to form of black holes. Then we could obtain most of parameters which have effect on this event.

First we considered the tension of loops as consent ($q = 0$) and for $J \neq 0$, we find a critical value of initial radii of loops as α, where if $r(t_0) < \alpha$ and $|J| < 1$ loops will collapse to form a black hole and for $r(t_0) > \alpha$ and $|J| > 1$ they will stay in universe,and vice versa. It is interesting the case of $|J| > 1$ by increasing J the critical value also increases. And these critical values are larger than L which means that larger than the size of universe .Thus we can conclude for larger angular momentum many of cosmic string loops stay in universe and avoid to become black hole.

Also we find that any change in the value of M can lead to a new critical initial radii,it means that by enlarging the value of the BTZ black hole mass ,the critical radii becomes larger and the possibility of forming black holes become lesser. We con point out for $M \leq 0$ without having any constraint all the loops will finally collapse to form of black holes.

Also we got to interesting results for time dependent tension of cosmic string loops. And find that for all values of $q \geq 0$ any variation of q has no effect on the value of the critical radii, then the results are the same as $q = 0$. But by increasing the positive values of q the loops take longer time to become black holes, see Fig (4). We have tried to find the effect of$q < 0$ on the evaluation of cosmic strings. In that case and by considering $J = 1$ there are two limits for q as ,$q < -4.78$ and $-.65 < q < 0$ where lops avoid to become black holes and will remain.

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